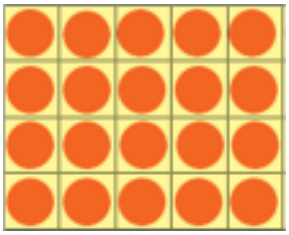
Lower Key Stage 2

# MULTIPLICATION

# X

## Developing Conceptual Understanding:
## CONTEXTUAL - LINGUISTIC - PRACTICAL - CONCRETE

- **Arrays:**
  Link arrays to calculating the area of rectangles (and commutative multiplication sentences):



Height (or length) = 4 squares

Base (or width) = 5 squares

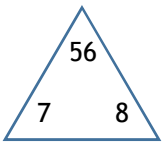
Area = base x height ***or*** length x width

= 5 x 4 ***or*** 4 x 5 = 20 squares

- **Commutativity:**
  Make use of fact families (and inverse operations knowledge) when learning tables:

  eg: 4 x 3 = 12 so 3 x ☐ = 12 ; 12 ÷ 4 = 3 ; so ☐ ÷ 3 = 4 etc

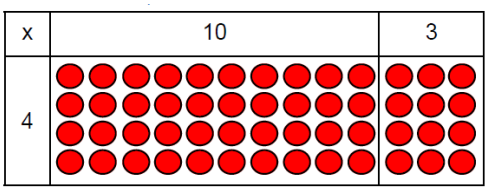
  Apply the Fact Family Triangle jotting from KS1 for learning key facts in x and ÷ :




7 x 8 = 56; 8 x 7 = 56; 56 ÷ 7 = 8; 56 ÷ 8 = 7

- **Doubling known facts:**
  Year 3 should revise the 5x and 10x tables facts, using doubling of multiples of 5 to derive multiples of 10; they should apply this strategy to derive the 4x tables facts by doubling multiples of 2 and the 8x tables facts by doubling multiples of 4 as they progress through the year.

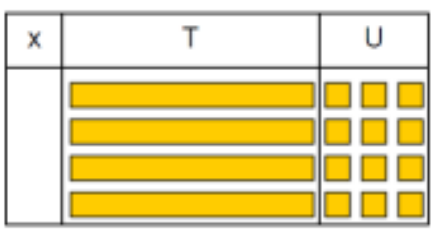
- **From Arrays to the Grid Method:**
  Scaffold the understanding of the grid method by showing the link with arrays:




13 x 4 = 4 rows of 13 (or 13 columns of 4)

The 13 can be partitioned into

10 + 3




This can be modelled with Base 10.

Adding up the Tens and the Ones gives:

4 x 13 = 40 + 12 = 52

This can be further progressed to the use of place value counters for larger numbers (see below).

- **Multiples:**
  The children must be familiar with, and use repeatedly, the language of **'multiple'** (as the answer to any two numbers that have been multiplied together - so 'multiples of 5' just mean the answer to any number multiplied by 5).

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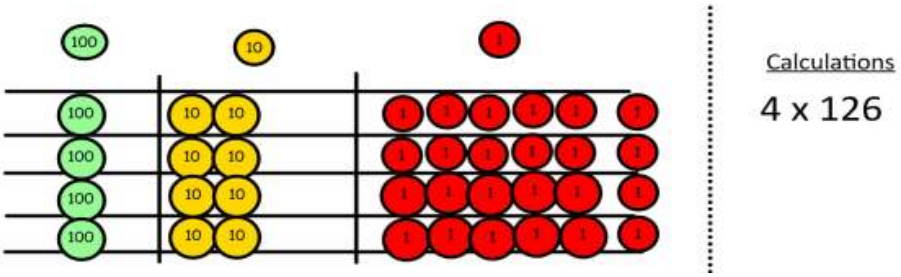
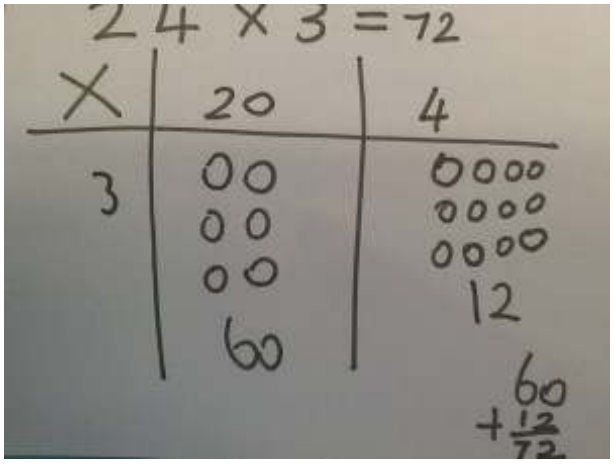
Developing Conceptual Understanding:
PICTORIAL – MENTAL

- **Grid method:**
  Intermediate steps include using place value counters (where different coloured counters are used to stand for a particular place value amount – as here with green = 100; yellow = 10; red = 1):




This can then progress to children recording their own counters pictorially:




- **Zero concept:**
  It is crucial children develop the concept of multiplying by 0:

  eg: 0 x 14 as 'No lots of 14 equals 0 (or nothing)'
  14 x 0 as 'fourteen lots of nothing = nothing'

  Any number multiplied by zero = 0. "Even a million lots of nothing is still nothing!"

- **Multiplying by 10 and 100:**
  Use place value to illustrate that:

- **when we multiply a number by 10, the digits move ONE place value column to the LEFT**
- **when we multiply a number by 100, the digits move TWO place value columns to the LEFT**

***NEVER* teach them to 'put a zero on the end of the number' to multiply by ten:**

**This is mathematically incorrect; has to be un-learned subsequently and is especially damaging in developing children's understanding of multiplication of decimals further up in KS2.**

- **Multiplying by 4:**
  To multiply by 4, you can double the number, then double the answer (ie 'double and double again')

  So, for 14 x 4 do double 14 = 28, then double 28 = 56 So 14 x 4 = 56

  This is a quick, efficient and useful mental strategy.

- **Counting Stick:**


A crucial tool in supporting children to learn to skip-count, understand and learn the sequences of multiples for:

YEAR 3: counting in 4's, 8's, 50's and 100's
revising 2x, 10x, 5x from KS1
teaching 3x, 4x, 6x and 8x

YEAR 4: counting in 6's, 7's, 9's, 11's, 25's and 1000's
revising 2x,10x, 5x, 4x, 8x, 3x and 6x tables from Year 3
teaching 12x (by doubling 3x and 6x facts)
9x (including kinaesthetic method on fingers), 11x and finally 7x tables

- **Multiplication by factors:**
  Teach as a strategy for simplifying calculations for tables they don't know yet, or to perform calculations mentally:

  Eg: x 6: 7 x 6 the factors of 6 include 3 x 2

  So, 7 x 6 is the same as 7 x 3 x 2, so do 7 x 3 = 21 and double the answer = 42

  Eg: x 20: 15 x 20 the factors of 20 include 2 x 10

  So, 15 x 20 is the same as 15 x 2 x 10, so do 15 x 2 = 30 and multiply 30 by 10 = 300

- **Multiplying by 3 numbers:**
  Continue to multiply three (or more) numbers – looking for ways to simplify sums using factors or known facts where possible:

  Eg: 7 x 5 x 4 — 5 x 4 = 20 so do 7 x 20, which is the same as 7 x 2 x 10
  So, double 7 and multiply the answer by 10 = 140

- JUST KNOW IT! YEAR 3

All 2x, 10x, and 5x tables x and ÷ facts (including odd and even numbers)
3x, 4x, 6x and 8x tables x and ÷ facts
Count in steps of 3, 4, 6 and 8
Double any two-digit number

- JUST KNOW IT! YEAR 4

All times tables facts to 12 x 12 in x and ÷
Count in steps of 7, 9, 11, 12
Double three-digit numbers (and two-digit decimals)
Mentally work out 10x and 100x a number

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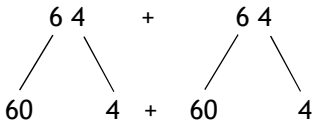
Abstract Recording:
INFORMAL JOTTINGS

- **Doubling by repeated addition:**

Eg: double 64:




60 4
+ 60 4
120 + 8 = 128

As column addition methods become consolidated, this would lead to:

Eg: double 96:

| H | T | O |
|---|---|---|
| | 9 | 6 |
| + | 9 | 6 |
| | 1 | 1 |
| 1 | 9 | 2 |

- **Towards the Grid Method (following on from arrays and bar models):**
  Start with multiplying by one digit numbers and setting out the addition clearly next to the grid to calculate the answer.

Example 1: Moving beyond times tables: **14 x 3**

| | 10 | 4 |
|---|---|---|
| 3 | 30 | 12 |

**So, 14 x 3 = 30 + 12 = 42**

Example 2: Before the child is secure with multiplying by multiples of 10: **24 x 5**

| | 10 | 10 | 4 |
|---|---|---|---|
| 5 | 50 | 50 | 20 |

**So, 24 x 5 = 50 + 50 + 20 = 120**

Example 3: Once the child is secure with multiplying by multiples of 10: **24 x 5** becomes

| | 20 | 4 |
|---|---|---|
| 5 | 100 | 20 |

**So, 24 x 5 = 100 + 20 = 120**

- **Grid Method:**

Example 3 in the bar/array format shown above, would be presented like this as a grid:

**24 x 5**

| X | 2 0 | 4 |
|---|---|---|
| 5 | 1 0 0 | 2 0 |

**NB: leave a blank square!**

Children should be taught to leave the blank square to allow for extra place value columns in the answer box – failure to do so is the most common cause of error in grid method working, as they try to squash too many digits into answer boxes that are too small and lose the place value columns as they add – leading to incorrect column totals.

EXPECTATION: By the end of Year 3, we would expect most children to be using the grid method to accurately calculate 2 digit by 1 digit multiplications.

- **Progressing the Grid Method:**

Multiplying by 2 digit numbers:

| | 10 | 8 |
|---|---|---|
| 10 | 100 | 80 |
| 3 | 30 | 24 |

Initially, go back to the array/bar model approach, to show that (as in this example);
13 x 18 can be partitioned into two bars – one for 10 x 18 and one for 3 x 18, stuck together.
This works because 13 lots of 18 is equal to 10 lots of 18 add 3 lots of 18.
[NB this also works as 18 x 13 = (10 x 13) + (8 x 13) if you follow vertical bars]

Once this is understood, grid method can be extended to bigger numbers x 2 digits:
Eg: 342 x 18

| x | 3 0 0 | 4 0 | 2 |
|---|---|---|---|
| 10 | 3 0 0 0 | 4 0 0 | 6 0 |
| 8 | 2 4 0 0 | 3 2 0 | 8 |
| | 5 4 0 0 | 7 2 0 | 6 8 |

```
  5 4 0 0
+   7 2 0
      6 8
    1
  6 1 8 8
```

Note: Children should use vertical, column addition to add each column separately (by simply continuing the grid down to create an answer box underneath) - hence the need for extra place value column space in each box of the grid.
By adding all these subtotals together at the end (in a separate column addition), they obtain the final answer.

Expectation: We would expect the majority of Year 4 children to be confidently using this informal jotting method by the end of that year.

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**x**

Abstract Recording:
OUR WRITTEN METHOD

During Year 4, once children have consolidated partitioning and are secure with the grid method informal jotting, they should begin to transition over to learning the expanded method of (long) multiplication.

- **<u>'Long' Multiplication method:</u>**
  Introduce alongside the grid method:

Eg: **35 x 7**

**Grid method:**

| x | 30 | 5 |
|---|---|---|
| 7 | 210 | 35 |

**= 245**

**Equivalent, 'Expanded' or 'long' multiplication method:**

```
  TO
  35
 x 7
  35   (7 x 5)
+210   (7 x 30)
 245
```

Eg: **127 x 6**

| x | 100 | 20 | 7 |
|---|---|---|---|
| 6 | 600 | 120 | 42 |

```
  HTO
  127
x   6
   42   (6 x 7)
  120   (6 x 20)
+600    (6 x 100)
  762
```

EXPECTATION: We hope to secure this expanded method for the majority of children by the end of Year 4; we will not begin to teach 'short' multiplication until this is secure (in practice, most children should be ready for this during the autumn term in Year 5)